Meaning of ratio (and connection to percent)

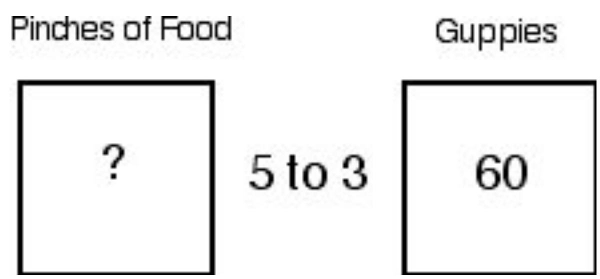
Many students have difficulty with understanding and applying ratio. One reason is because instructional practice tends to associate ratio with fractions almost immediately. This is not wise. Fractions are already troublesome for students. Linking ratio, a numerical relationship between two quantities, fairly immediately to fraction is unlikely to help students understand ratio well. This is especially the case if they understand fraction solely as "cutting up pizzas".

A better way to develop ratio is to first develop it as ‘a rule that controls quantities in containers’. This requires that students understand the ‘groups of’ meaning of multiplication and have some multiplication skills. At a later point, ratio should be connected to fractions.

The container model of ratio can provide students with a visual framework for understanding ratio and a conceptual tool for solving ratio problems. Consider this problem.

> **When Harry feeds his guppies, he has to put five pinches of food on the surface of the water for every three guppies. If Harry has 60 guppies, how many pinches of food should he put on the water?**

The container model for representing what is going on in the problem would look as shown here. Representing ratio in this way can help students identify the structure of a problem, something that is important when solving routine problems. The calculation of the answer by using an arithmetic method is finishing touch.



What follows is a description of a part of my case study research on how students learn mathematics and what kind of mathematics they can learn. The students in the study regularly met with me on Saturday mornings for a period of 5 years (from grades 1 to 5), learning mathematics by discussion and problem solving (a problem-solving climate of learning). The description concerns using the container model of ratio to develop three grade 5 students’ understandings of ratio and percent and their problem-solving skills with ratio and percent.

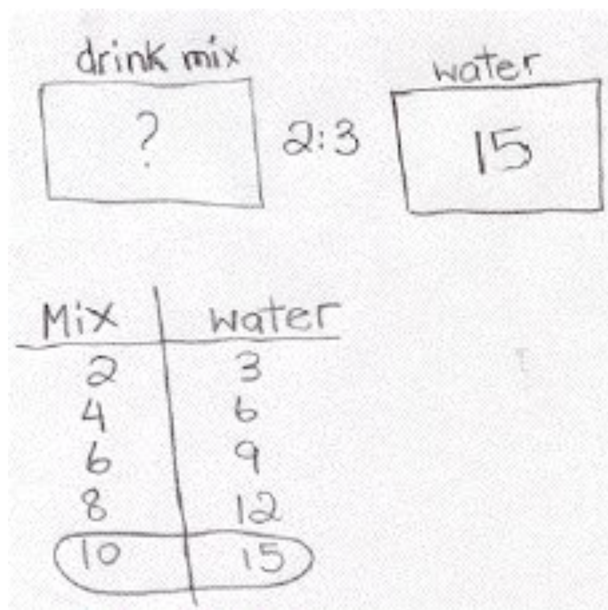
During one Saturday session, a student asked, “*What is percent?*” This stimulated the others to become interested in it as well. I could have used a conventional approach involving percent as fraction to address the question but did not. Relating percent immediately to fraction has issues that concern the functional use of percent to solve problems (problem solving should be central rather than translating between notational systems) and that concern students’ confusions about fractions. Developing percent through ratio can address these issues. Accordingly, my response was "*Can you wait a bit before I answer your question directly? There is something you need to understand first*." The students accepted the delay. They were used to such responses.

The students seemed ready for ratio. They understood multiplication as 'groups of' and were proficient with mental multiplication to 10 x 10. I created a teaching plan spread over four Saturdays that involved developing ratio through the container model, solving ratio problems, using ratio to develop percent (e.g., 7% as a ratio of 7 to 100), and solving simple percent problems.

I began the development of ratio with a discussion about making juice. We examined frozen fruit concentrate containers and discussed the instructions. We made a double batch of raspberry juice by following the instructions - one part concentrate to three parts water. After a juice break, I gave the students a powdered drink mix label for making strawberry juice with the instructions on it, "*Mix 2 tablespoons of powder with 3 cups of water.*", and the following problem.

> **You want to make a lot of strawberry juice for a birthday party and decide 15 cups of water is needed to make it. How many tablespoons of drink mix powder should you add to the water?**

They discussed the problem and solved it by making two columns of numbers and using addition. They added 2 to the drink mix column for every 3 to the water column until 15 was reached in the water column. The number of tablespoons, 10, was found across from 15.



I could have left the matter of ratio at this intuitive addition stage but that would not have served the students well in the long run. Studies have shown that one reason why students have difficulty solving routine problems is that they do not analyze a problem situation by means of a conceptual model, a model that provides a basis for identifying what is going on in the problem. Rather, they think primarily in terms of arithmetic as was the case with the students' addition method. To address this, I presented the students with my solution to the drink mix problem, a solution that involved the container model of ratio as illustrated in the diagram.

We discussed how the notation, 2:3, in the model was a short way of saying the ratio rule: 'for every 2 tablespoons we need 3 cups'. We also discussed why the labeled containers were helpful for thinking about similar problems. The conclusion we reached was that the containers with the ratio rule sitting in between them provided a picture of what the problem was about. The first session ended with the students solving three more ratio problems (for contexts of making juice, expanding a recipe, and making glue). I asked them to use the container model to show what the problem was about but they could use any method they wanted to calculate the answer. Not surprisingly, they used the addition method.

The second session began with a ratio problem in order to assess whether the students could still use the container model. Because they could, it was time to provoke some "uncomfortableness" about their addition method for calculating answers. Using addition initially is useful but it obscures the multiplicative aspect of ratio.

I presented the students with the following problem.

> **You need to make a lot of goop to use for preparing a canvas surface for painting. The recipe for the goop requires 2 cups of flour for every 3 cups of water. You decide to use 420 cups of flour to make enough goop. How many cups of water will you need to add if you are going to follow the recipe?**

The students began solving the problem by representing what was going on it with the container model of ratio. Questions and comments soon emerged.

- Student #1: *It's going to take a lot of adding to get to 420.*
- Student #2: *420 is too big for adding.*
- Student #3: *Do you really want us to figure this out?*

I asked if there might be a better way than adding to calculate the answer. They thought there should be one but were not sure about what it might be. To help develop a multiplication method for calculating the answer, I presented them with a numerically simpler version of the problem.

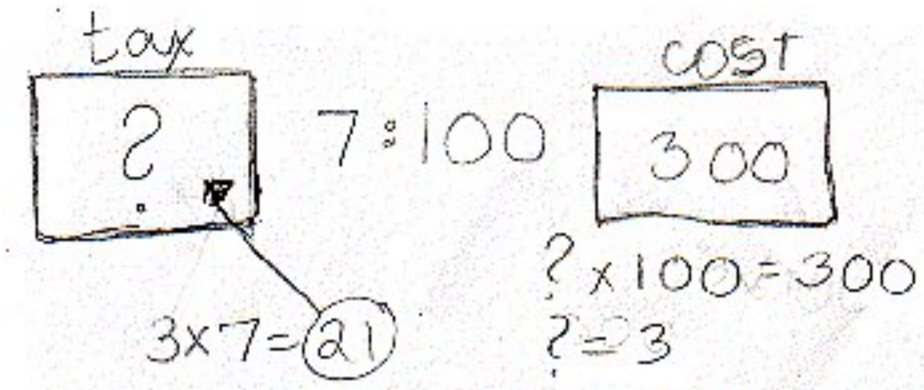
> **You decide to use 12 cups of flour. How much water is needed? I suggested that they think about the number of groups of 2 needed to reach 12 cups of flour and to use that thinking to figure out the number of cups of water needed.**

The third session began with the students solving routine problems. Some problems involved ratio; others involved addition or multiplication. I asked them to represent what was going on in each problem by using the container model or a number sentence, which ever seemed appropriate. It was important to find out if they could identify what was going on in a problem when faced with different types of problems. If they could not, this would indicate that their problem-solving skills were limited to circumstances where all of the routine problems concerned the topic of the day. This kind of problem-solving skill is not useful when confronted with problems that can come at you helter-skelter in the world outside of the classroom. The students were able to identify what was going in the problems and to solve them successfully.

For the last part of the session, we began the development of percent. We used word analysis to arrive at the conclusion that percent had something to do with 100. The words 'cent' and 'century' helped with that.

I told the students that 7% (the sales tax in our province) was another way of describing a ratio of 7 to 100. Telling can be appropriate when students are interested in and have a sufficient conceptual basis for understanding the matter-at-hand.

The session ended with the students using the container model to solve three sales tax problems (see diagram for an example). The numerical complexity was restricted to purchases that involved multiples of 100.



Session four was a problem-solving festival. The students solved routine problems of varying types: ratio but not percent, addition, subtraction, multiplication, division. Numerical complexity was restricted to whole numbers because the students did not yet understand decimals and decimal arithmetic. The calculator could have been used for doing decimal arithmetic but that would have brought "magic" into the session. With only an occasional minor mishap, the students successfully identified what was going on in each problem and calculated the answer correctly. Because they were able to identify ratio in varied problems, this was evidence they understood it. It was also clear that they could apply ratio to percent.

At a later time when fraction equivalence, fraction arithmetic, and the ratio meaning of fraction are understood the students should be ready for learning about proportion as an equality of two fractions, (a grade 8 outcome). The container model of ratio can serve as a bridge for that where the ratio rule is associated with one fraction and the quantities in the two containers with the second fraction. In the case of the percent problem above, the proportion statement would look like: x/300 = 7/100.

This reading is a modified excerpt of an article published in delta-K (J. Ameis, Spring 2002).